Digital Filters
C.R. Nippert

There are many types of digital filters. The simplest digital filter uses the equation:

$$O = f \cdot I + (1-f)O_{last}$$

where

I = the input (the value to be filtered)
O = the filtered output
O_{last} = the last value of the filtered output
f = the filter constant (1=no filter, 0=complete filtering [the filter always returns the a constant value])

This digital filter is related to the ordinary first order lag

$$\tau_f \frac{dO}{dt} + O = I$$

where τ_f = the lag time.

The formula for the digital filter can be derived from the first order lag. Assume that the filter input, I, is constant over some small interval Δt. Then, rearrange the differential equation and integrate over the interval 0 to Δt.

$$\int_{O_{last}}^{O} \frac{dO}{I-O} = \int_{0}^{\Delta t} \frac{dt}{\tau_f}$$

$$\ln\left(\frac{I-O}{I-O_{last}}\right) = -\frac{\Delta t}{\tau_f}$$

Rearranging to solve for O

$$O = \left(1-e^{-\Delta t/\tau_f}\right)I + e^{-\Delta t/\tau_f}O_{last}$$

To obtain the digital filter, simply define

$$f = 1 - e^{-\Delta t/\tau_f}$$

Notice that $0 \le \tau_f < \infty$ leads to the conclusion that $0 \le f \le 1$. By examining the filter equation, we find that when f = 1 the input is simply passed through the filter. When f = 0 the filter ignores the input and merely outputs a constant value.

Tuning

Tuning can best be done based on tf. Marlin[1] recommends that $5/\omega_n < \tau_f < 0.05(\theta_p + \tau_p)$

Where ω_n = the noise frequency (normally, the sampling frequency)

[1] Thomas E. Marlin, Process Control 2nd ed. McGraw Hill, 2000, New York. P. 392

Example:

Calculate the range of values of f for a process for which the sample time, Δt, is 0.5 sec and the FOPDT constants are $K_p = 3$, $\tau_p = 20$ sec and $\theta_p = 15$ sec

Solution:
The lower limit for tf is 2.5 sec (5 times 0.5 sec). This corresponds to f = 0.181. The upper limit of tf is 1.75, corresponding to f = 0.249. The exact value depends on the amount of noise

Warning

The filter equations is sometimes written as

$$O = (1 - A) \cdot I + A \cdot O_{last}$$

For this form A = 1 –f